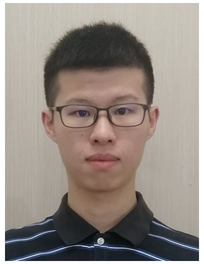
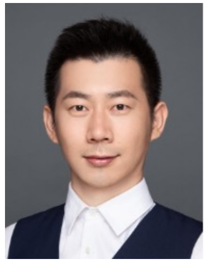
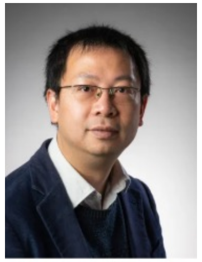
Ex-Ante Truthful Distribution-Reporting Mechanisms







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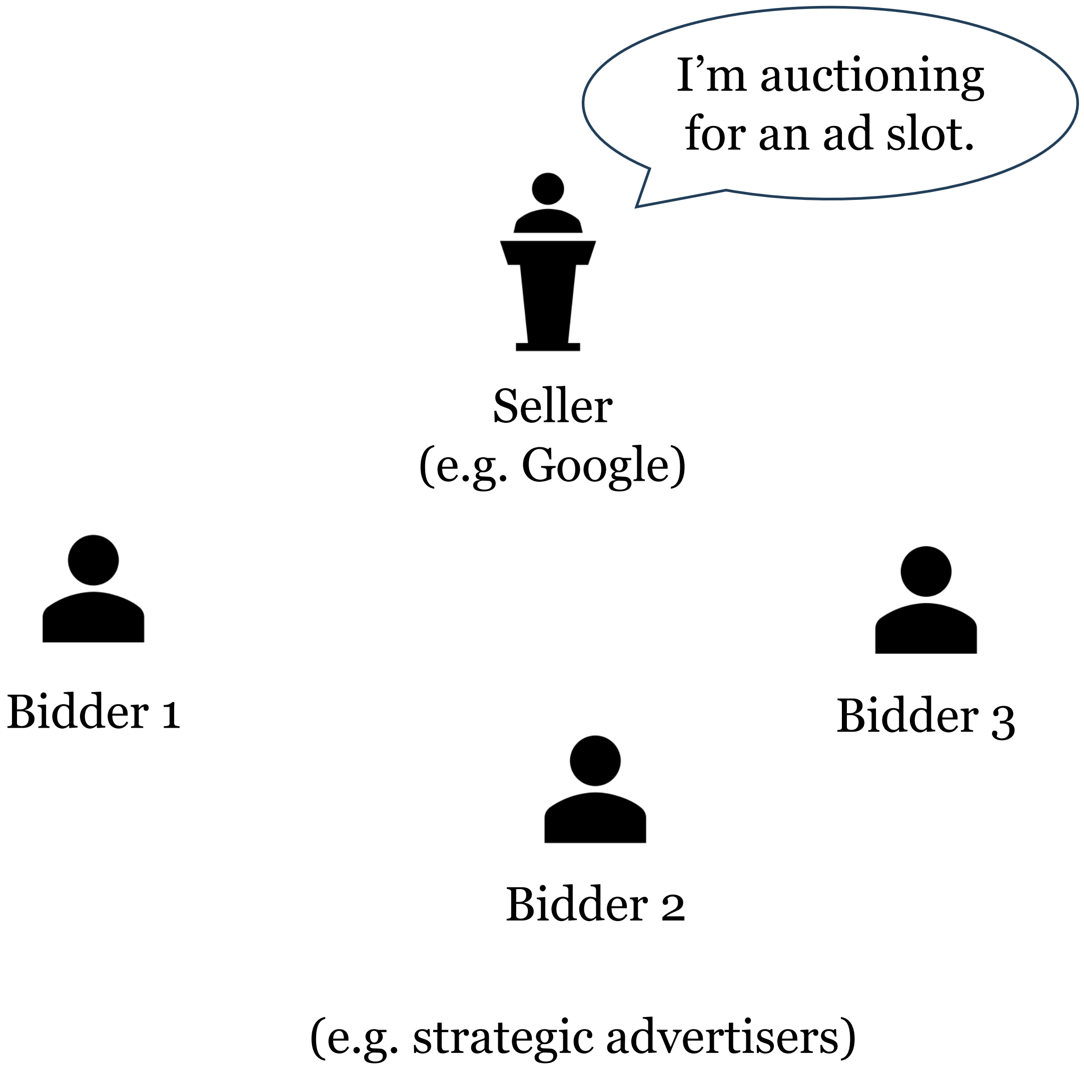
Harvard EconCS Seminar, 2026/5/8

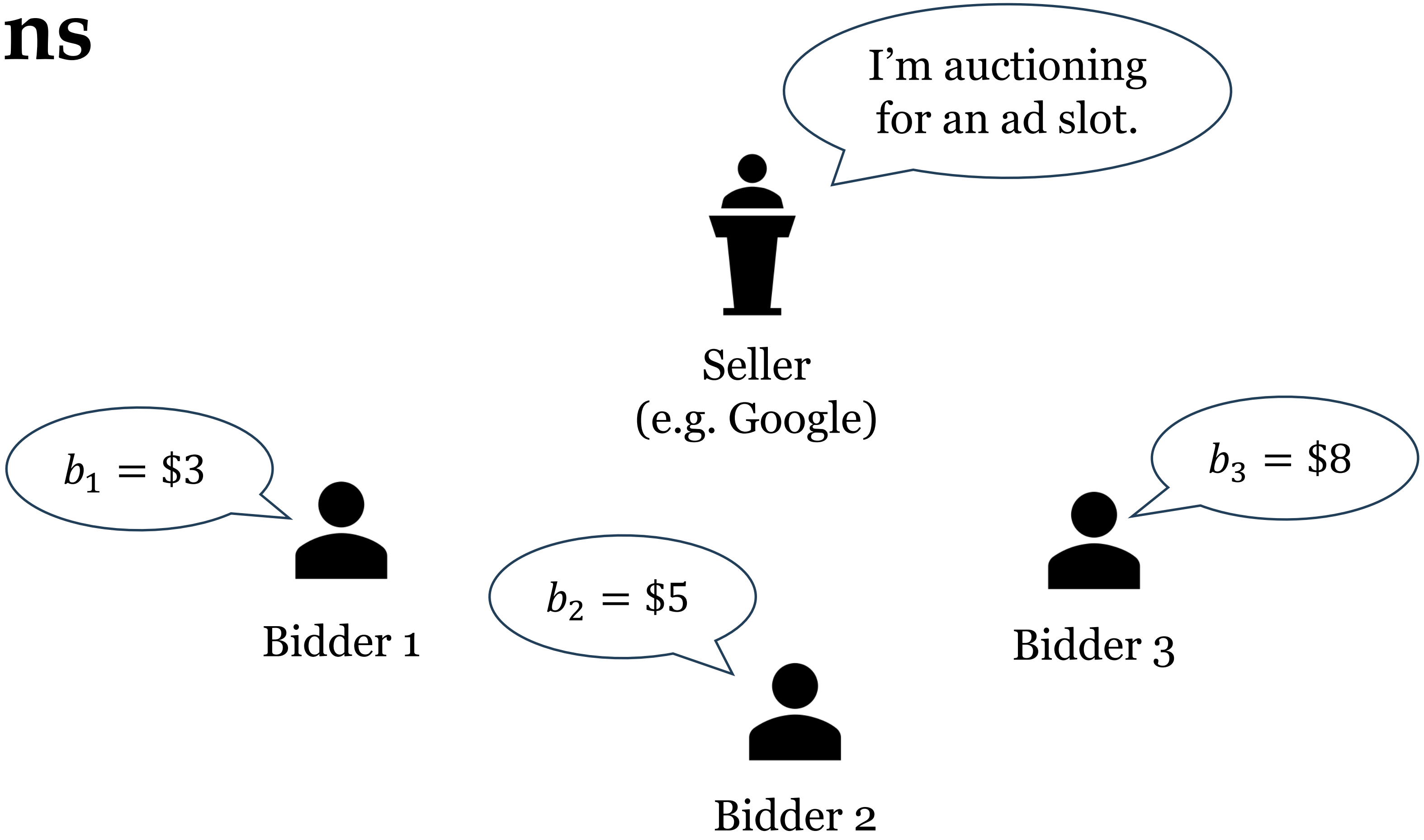
Outline

1. Background: Auctions
2. Myerson assumes known prior
3. We study unknown prior setting
4. We design *distribution-reporting* mechanism
    - Truthful
    - Maximize revenue

Auctions


I'm auctioning for an ad slot.
Seller
(e.g. Google)
Bidder 1
Bidder 3
Bidder 2
(e.g. strategic advertisers)

Auctions



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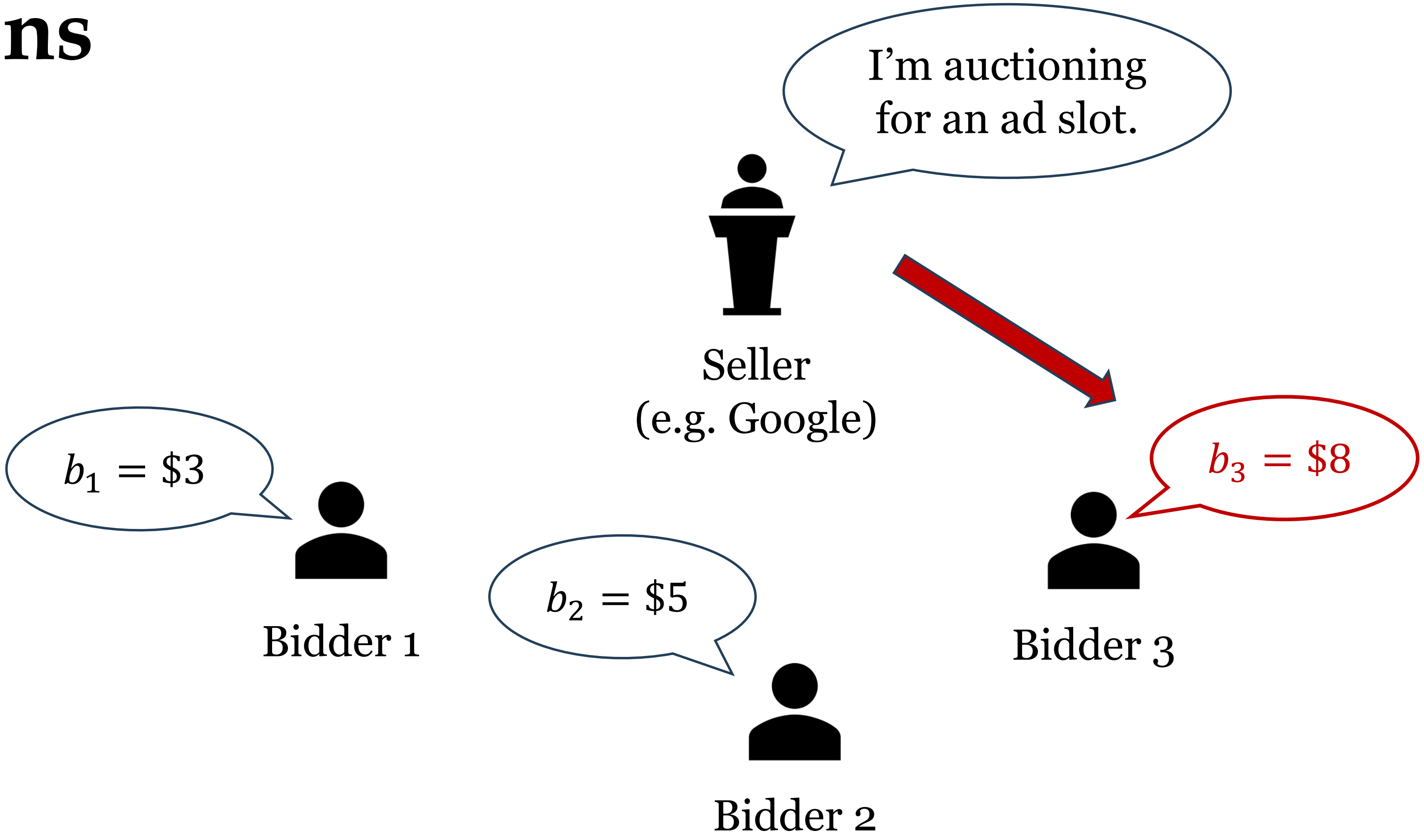
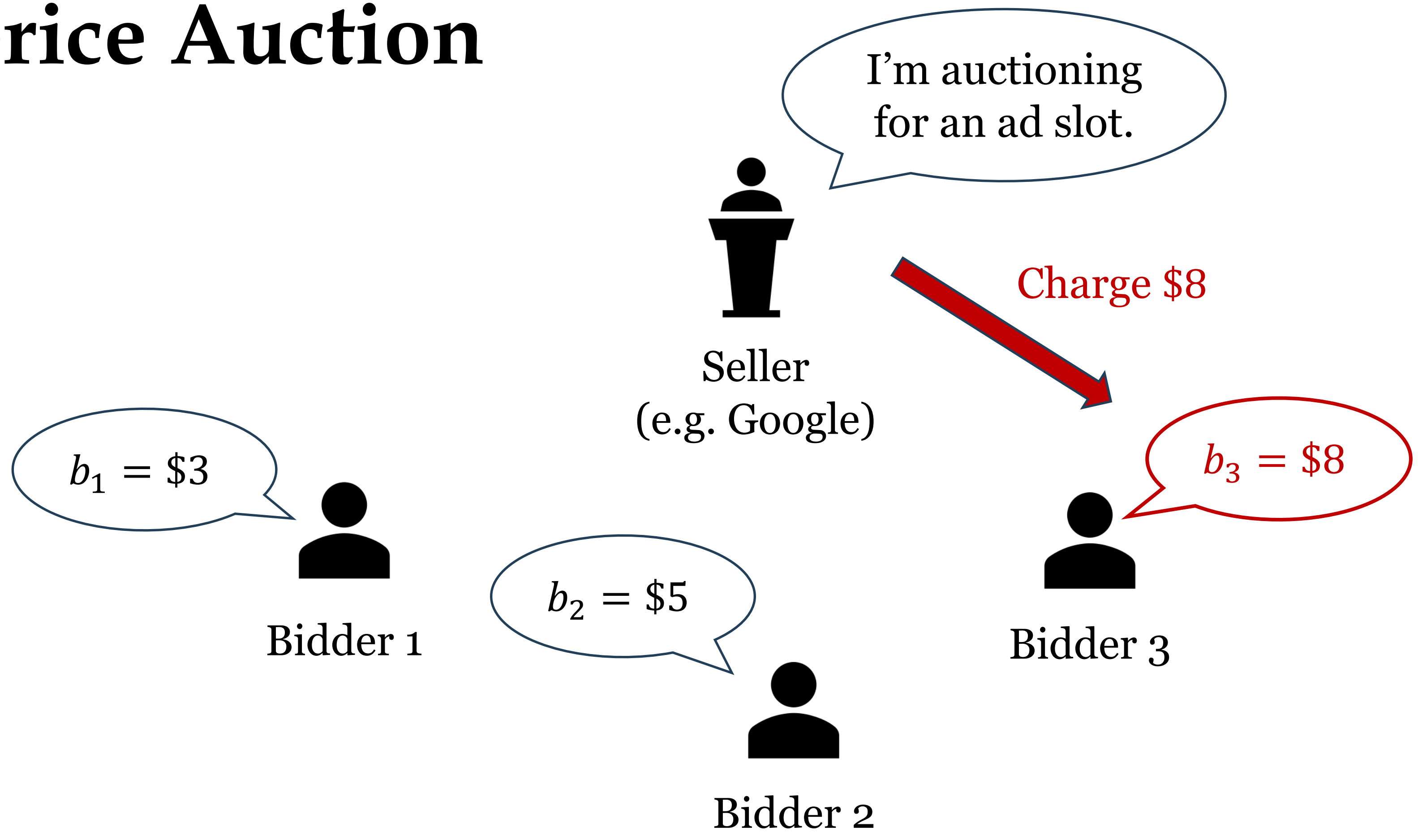
Auctions



(e.g. strategic advertisers)

First-price Auction



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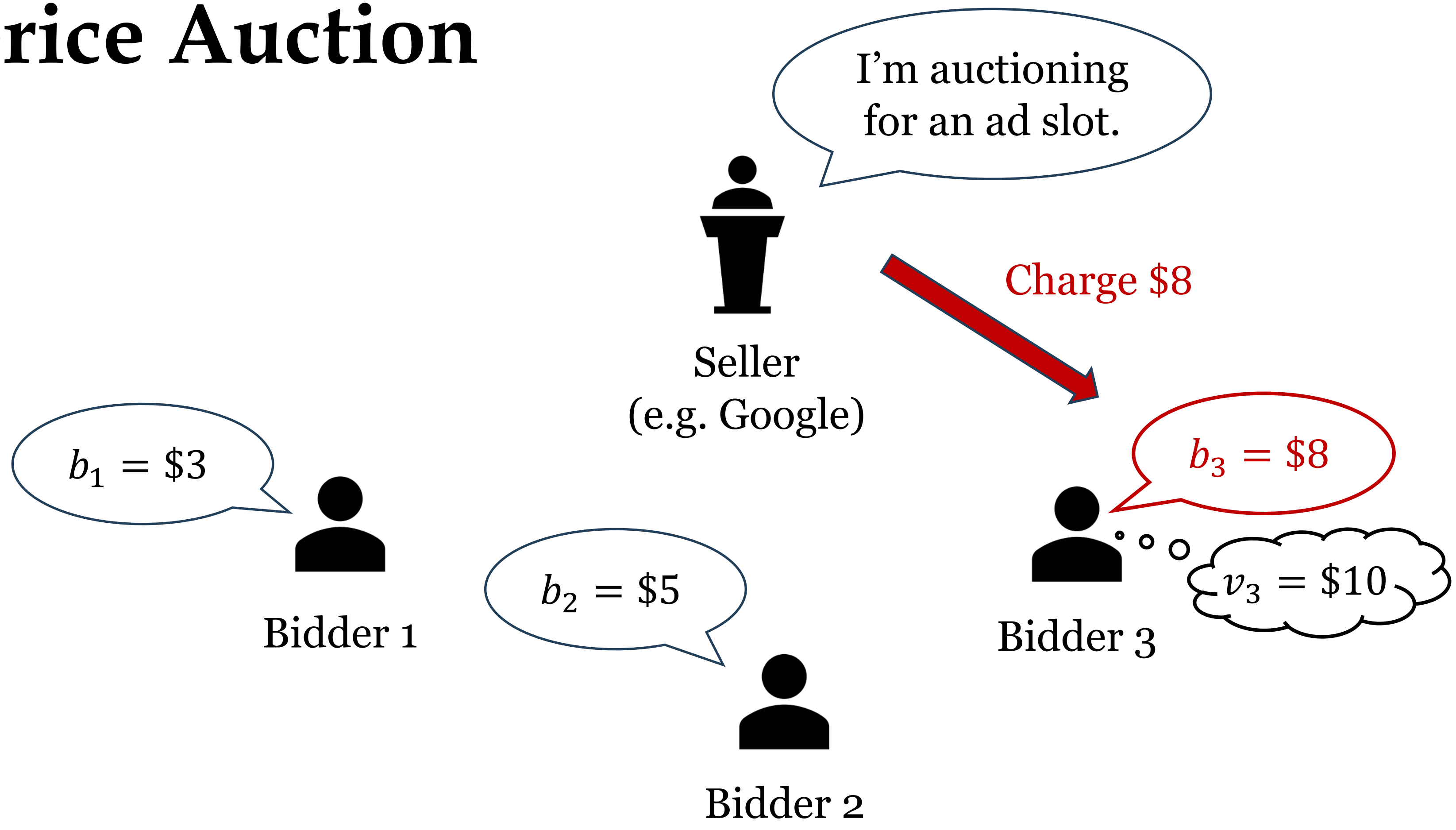

Issue: A bidder has an incentive to bid below her true value ($b_i < v_i$)

Second-price Auction

- Highest bidder wins
- Charged second-highest bid
- E.g.
  - $b_1 = \$3, b_2 = \$5, b_3 = \$10$
  - bidder 3 pays \$5

Second-price Auction

- Highest bidder wins
- Charged second-highest bid
- E.g.
  - $b_1 = \$3, b_2 = \$5, b_3 = \$10$
  - bidder 3 pays \$5
- Second-price auction is truthful: bidding true value $b_i := v_i$ is optimal

Formal Definition of Utility, Truthfulness

- Utility of bidder i =
  - $v_i - p_i$, if she wins and pays p_i
  - 0, otherwise
- A bidder would bid to maximize her utility

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Say a mechanism is Truthful / Incentive Compatible (IC), if $b_i := v_i$ maximizes utility, for every realization of other bids b_{-i}.

Valuation Model

- Bidder i's valuation v_i is drawn as $v_i \sim F_i$, independent
  - F_i is a commonly known distribution, but realized v_i is private

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Say a mechanism is Bayesian Incentive Compatible (BIC), if $b_i \coloneqq v_i$ maximizes expected utility, taken over other bids b_{-i} according to common knowledge.

Under **Truthfulness / IC** (make bidders bid $b_i = v_i$),

how to maximize expected **revenue** (money collected)?

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how to maximize expected **revenue** (money collected)?

- In the known-prior setting (seller knows all F_i), Myerson Auction (1981) characterizes the revenue-optimal auction.
- Myerson Auction = Second-price auction + reserve price

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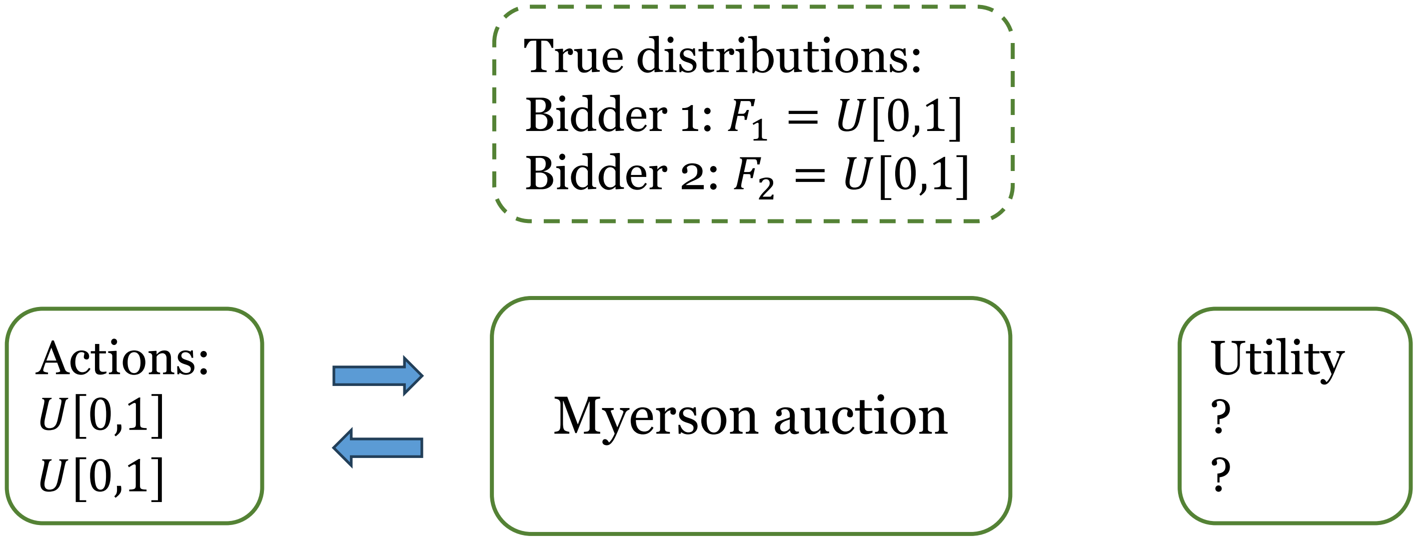
Q: Can we still use Myerson auction?

A: No, Myerson is no longer truthful under this framework [TZ18].

[TZ 18] Pingzhong Tang and Yulong Zeng, "The Price of Prior Dependence in Auctions", EC 18

Example: Myerson Auction is no longer truthful

Compare **standard setting** and this new setting



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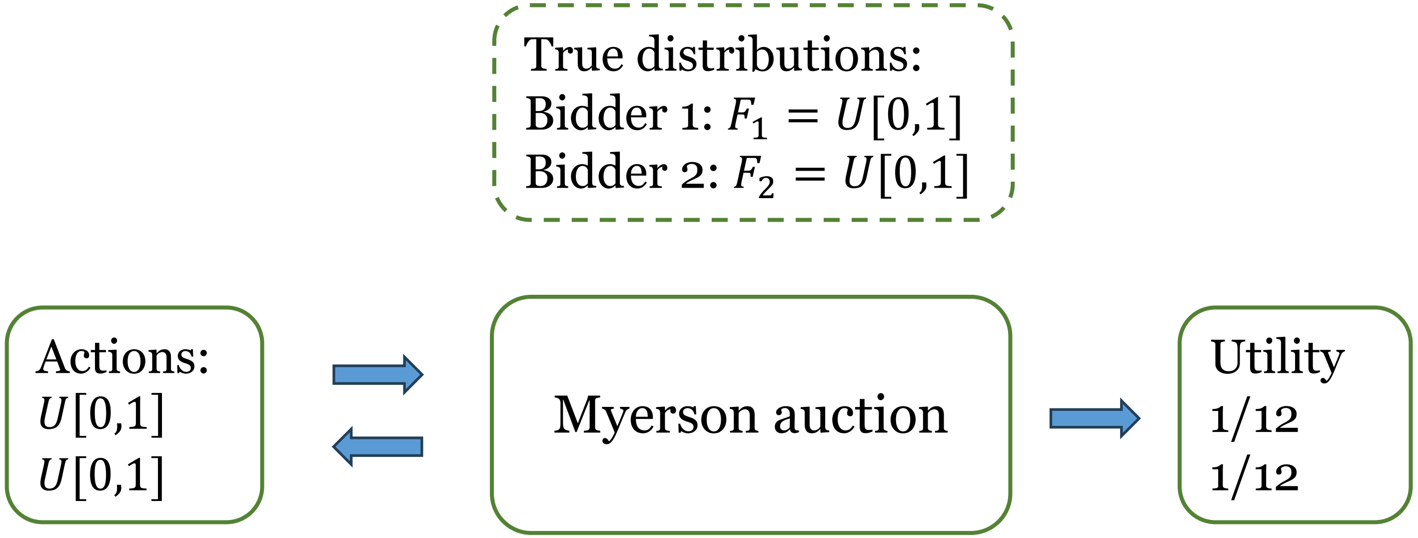

- Second-price auction with reserve price 1/2
- If $v_1 > v_2$ and $v_1 > 1/2$, allocate to bidder 1 and charge $\max(v_2, 1/2)$

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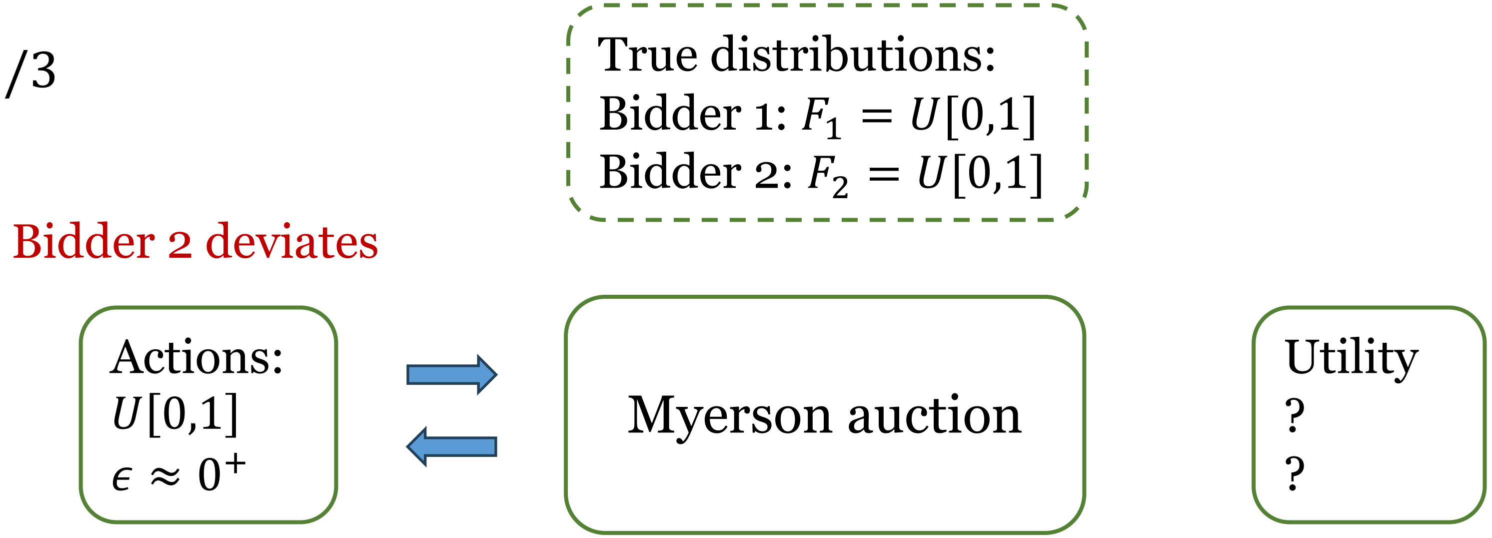
Compare standard setting and **this new setting**

Page 1/3



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Compare standard setting and **this new setting**

Page 1/3

True distributions:
Bidder 1: $F_1 = U[0,1]$
Bidder 2: $F_2 = U[0,1]$

Bidder 2 deviates

Actions:
$U[0,1]$
$\epsilon \approx 0^+$

Myerson auction

Utility
?
?

- Bidder 2 wins if $2v_1 - 1 < \epsilon$, or $v_1 < \frac{1}{2}(1+\epsilon)$
- $\mathrm{E}[u_2] \approx \int_0^1 \frac{1}{2}(1+\epsilon)(v_2 - \epsilon)dv_2 \to \frac{1}{4}$

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Page 2/3

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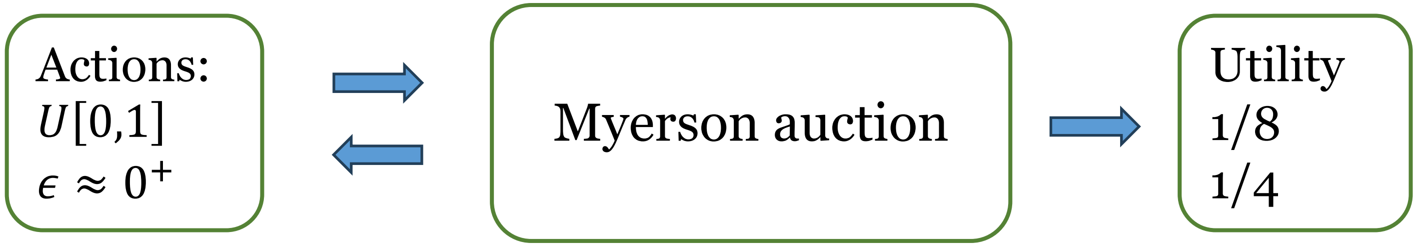

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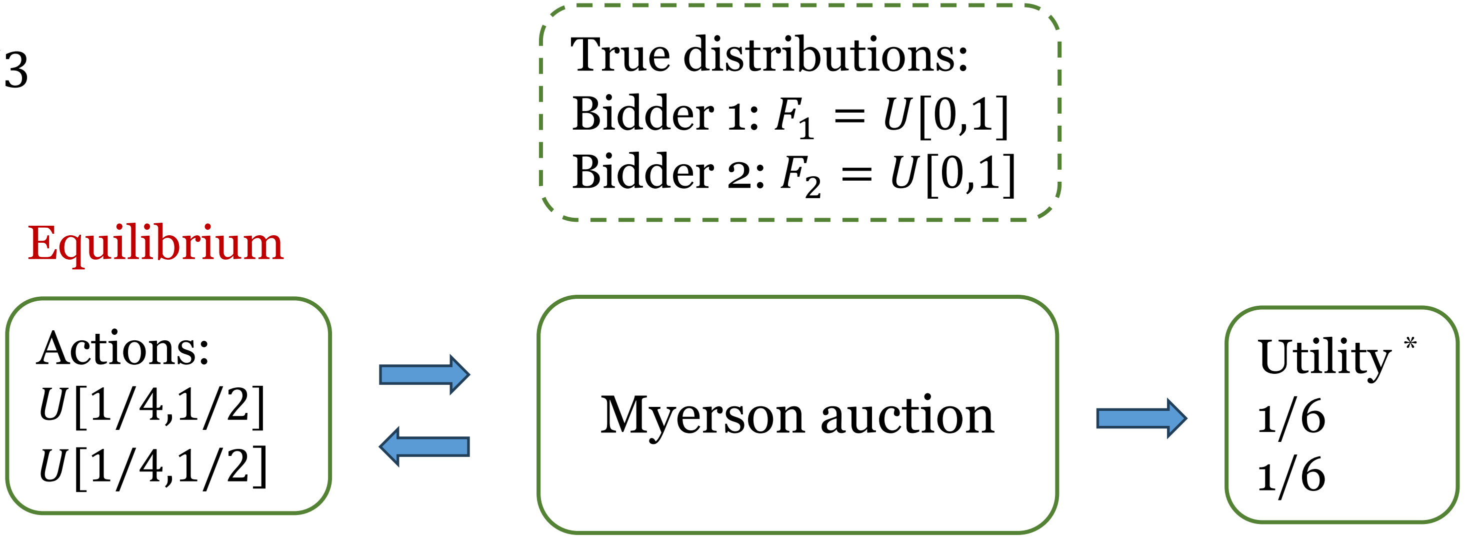

* Myerson becomes equivalent to running first-price auction on $U[0,1], U[0,1]$

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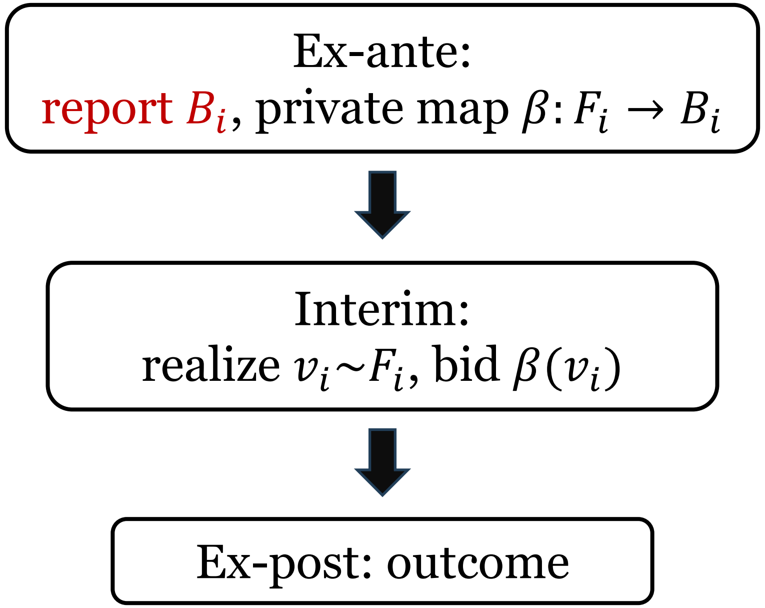
Formalization of Distribution-Reporting Model

- Single-round game, 3 stages
- Let bidder i report a bidding distribution B_i at ex-ante stage



Re-examine the Definition of Truthfulness

Ex-ante:
report B_i, private map $\beta: F_i \to B_i$

↓

Interim:
realize $v_i \sim F_i$, bid $\beta(v_i)$

↓

Ex-post: outcome

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Standard truthfulness **(Bayesian IC)** is insufficient: bidders can misreport $b_i \sim \boldsymbol{B_i} \neq \boldsymbol{F_i}$ to **manipulate** mechanism rules [TZ18]

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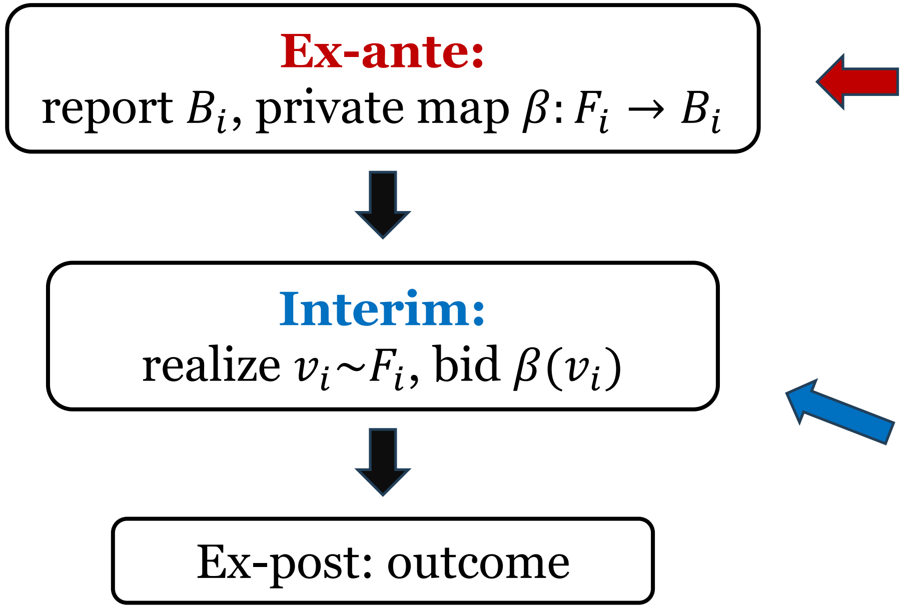

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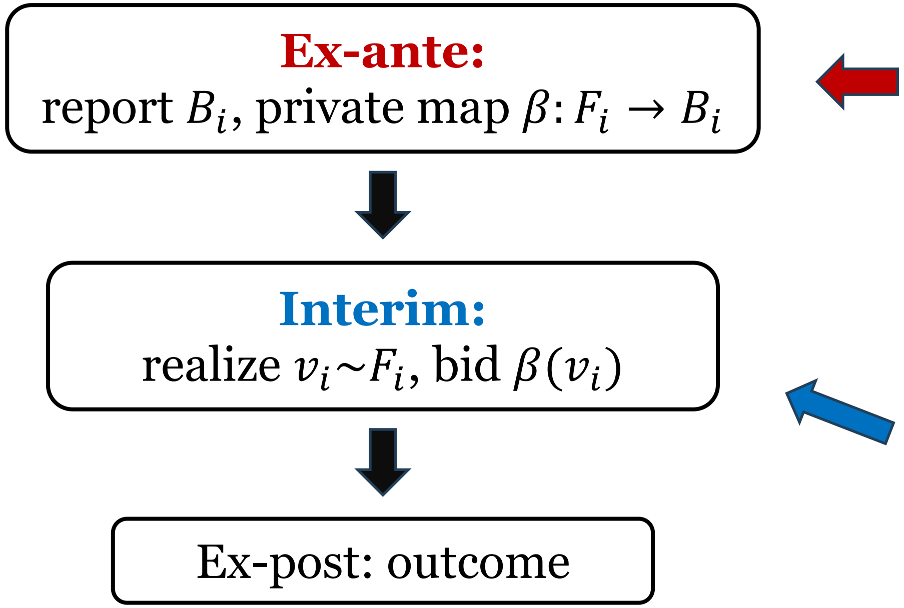

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Why is it useful?

Standard truthfulness **(Bayesian IC)** is insufficient: bidders can misreport $b_i \sim \boldsymbol{B_i} \neq \boldsymbol{F_i}$ to **manipulate** mechanism rules [TZ18]

Under **ex-ante IC** (make bidders report $B_i = F_i$),

can we guarantee *worst-case* **revenue**?

Main Result (informal)

We design an ex-ante IC mechanism achieving **an either-or guarantee**

$$\min\left(\lambda_1 \cdot \mathrm{Wel}(\boldsymbol{F}), (1+\lambda_2) \cdot \mathrm{Rev}(\mathrm{SPA}; \boldsymbol{F})\right)$$

that is **optimal** up to constant factors.

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that is **optimal** up to constant factors.

- Wel is welfare (max realized value) = $\mathrm{E}_{v\sim F}[\max_{i\in[n]} v_i]$
- Rev(SPA) is the revenue of second-price auction = $\mathrm{E}_{v\sim F}[\operatorname{secmax}_{i\in[n]} v_i]$
- λ_1 and λ_2 capture the trade-off between Wel and Rev(SPA)

Mechanism: SPA + ex-ante entry fee

$$\mathrm{Wel} = \mathrm{E}_{v \sim F}[\max_{i \in [n]} v_i]$$
$$\mathrm{Rev(SPA)} = \mathrm{E}_{v \sim F}[\operatorname{secmax}_{i \in [n]} v_i]$$

- Start with SPA (or any IC bid auction)
- Compute bidder i's expected utility under SPA
  - If utility ≥ threshold $\tau_i(F_{-i})$: let them play SPA but charge τ_i
  - Otherwise: exclude them (but still *simulate* their bid inside SPA)
- Let $\tau_i(F_{-i}) := \alpha \cdot \mathrm{E}[\max_{j \neq i} v_j]$.
- Randomize α over $\approx O(\log n + K)$ scales.

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- Why truthful?
- Why potentially optimal?

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- Why truthful?

  τ_i only depends on others' reports.
- Why potentially optimal?

  [TZ18] SPA (prior-independent) can weakly outperform several prior-dependent truthful auctions.

Main result (formal)

$$\text{Wel} = \mathrm{E}_{v\sim F}[\max_{i\in[n]} v_i]$$

$$\text{Rev(SPA)} = \mathrm{E}_{v\sim F}[\operatorname{secmax}_{i\in[n]} v_i]$$

Either-or guarantee: $\text{Rev}(M; \boldsymbol{F}) \geq \min\big(\lambda_1 \cdot \text{Wel}(\boldsymbol{F}), (1+\lambda_2)\cdot \text{SPA}(\boldsymbol{F})\big), \ \forall \boldsymbol{F} \in \boldsymbol{\mathcal{F}}$

- **Positive.** For any $n \geq 2$ bidders and any $K \geq 1$, there **exists** an ex-ante IC distribution-reporting mechanism that guarantees an expected revenue of

$$\min\left(\frac{1}{24(K+\log_2 n)} \cdot \text{Wel}(\boldsymbol{F}), 2^K \cdot \text{Rev}(\text{SPA}; \boldsymbol{F})\right)$$

- **Negative.** For any $n \geq 2$ bidders and any $K \geq 6$, **no** ex-ante IC distribution-reporting mechanism can always guarantee an expected revenue of

$$\min\left(\frac{64}{K+\log_2 \text{n}} \cdot \text{Wel}(\boldsymbol{F}), 2^K \cdot \text{Rev}(\text{SPA}; \boldsymbol{F})\right)$$

Questions?

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